Algebraic Geometry

II. Cohomology of coherent sheaves

Ulrich Görtz
Universität Duisburg-Essen
Fakultät für Mathematik
45117 Essen
ulrich.goertz@uni-due.de

Torsten Wedhorn
Technische Universität Darmstadt
Fachbereich Mathematik
Schlossgartenstraße 7
64289 Darmstadt
wedhorn@mathematik.tu-darmstadt.de

Contents

Introduction **1**

17 Differentials **5**
Differentials for rings and extensions of algebras 5
Differentials for sheaves on schemes 10
The de Rham complex 24
Exercises 29

18 Étale and smooth morphisms **31**
Formally unramified, formally smooth and formally étale morphisms 31
Unramified and étale morphisms 39
Smooth morphisms 48
Exercises 63

19 Local complete intersections **68**
The Koszul complex and completely intersecting immersions 68
Local complete intersection and syntomic morphisms 81
Exercises 87

20 The étale topology **91**
Henselian rings 92
The étale topology 101
The étale fundamental group of a scheme 115
Exercises 142

21 Cohomology of $\mathcal{O}_X$-modules **151**
Categories of abelian sheaves and of $\mathcal{O}_X$-modules 153
Cohomology and derived direct image 158
Čech cohomology 178
Derived inverse image, Hom sheaves, and tensor products 187
Relations between derived functors 199
Perfect and pseudo-coherent complexes 210
Exercises 225

22 Cohomology of quasi-coherent modules **232**
Cohomology of quasi-coherent modules and Čech cohomology 233
Derived categories of quasi-coherent modules 243
Finiteness properties of complexes on schemes 255
Projection formula, base change and the Künneth formula 269
Exercises 288

23 Cohomology of projective and proper schemes **297**
Cohomology of projective schemes . . . 298
Coherence of higher direct images for proper morphisms . . . 306
Numerical intersection theory, Euler characteristic, and Hilbert polynomial . 314
The Grothendieck-Riemann-Roch theorem . . . 332
Cohomology and base change . . . 342
Hilbert polynomials and flattening stratification . . . 361
Exercises . . . 367

24 Theorem on formal functions **376**
Derived Completion . . . 376
The theorem of formal functions . . . 390
Stein factorization . . . 398
Algebraization . . . 411
Exercises . . . 431

25 Duality **435**
The right adjoint $f^\times$ of Rf_* . . . 436
Computation of $f^\times$ in special cases . . . 456
The functor $f^!$ . . . 463
Dualizing complexes . . . 469
Dualizing sheaves . . . 483
Duality for schemes over fields . . . 493
Applications to algebraic surfaces . . . 500
Exercises . . . 509

26 Curves **513**
Basic notions . . . 513
The Theorem of Riemann-Roch . . . 534
Special classes of curves . . . 547
Vector bundles on curves . . . 571
Further topics . . . 584
Exercises . . . 598

27 Abelian schemes **603**
Preliminaries and general results about group schemes . . . 604
Definition and basic properties of abelian schemes . . . 635
The Picard functor . . . 643
Duality of abelian schemes . . . 658
Cohomology of line bundles on abelian schemes . . . 692
Exercises . . . 734

F Homological Algebra **737**
Addenda to the language of categories . . . 737
Additive and abelian categories . . . 745
Complexes in additive and abelian categories . . . 754
Spectral sequences . . . 763
Triangulated categories . . . 772
Sign conventions . . . 780

Derived categories 781
Derived functors 791
Exercises 818

G Commutative Algebra II **826**
On regular and Cohen-Macaulay rings 826
On injective modules and Gorenstein rings 829

Bibliography **833**

Detailed List of Contents **846**

Index of Symbols **857**

Index **860**